Understanding Signal Theory through Play

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***Abstract*—Many undergraduate students find it difficult to learn, understand and conceptualize some of the the basic topics of signal processing and analysis theory such as: analog and digital signals, frequency domain, Fourier Transform, Discrete Fourier Transform, Nyquist-Shannon sampling theorem, aliasing etc. This paper presents an effective method to introduce some of the most important topics in an introductory Signal Theory course. A set of laboratory exercises is developed, where students experiment with several basic and advanced signal analysis and processing techniques, almost through game and play. The results of a poll show that students find considerable improvement in course organisation and greater usefulness of laboratory exercises in understanding course material.**

***Keywords*—signal processing, education, laboratory, students**

I. Introduction

Signal processing and analysis are one of the most important fundamental skills of a modern electrical engineer. Almost every electrical engineering faculty in the world today has some sort of a signal processing course, which deals with topics such as: Analog and discrete signals, Fourier series, Spectral analysis, Fourier transform, Discrete Fourier Transform, Nyquist-Shannon sampling theorem, aliasing etc. Faculty of Electrical Engineering Sarajevo, in a Signal Theory course on third semester (Department of Telecommunication) and Signal and System Analysis course on fifth semester (Department of Automatic Control and Electronics), teaches the basic and advanced concepts of signal processing and analysis.

Learning, and more important, understanding signal processing theory has proven to be not an easy task for students. Signal processing concepts almost always have a precise and clear mathematical background, but often students find difficult to understand how a particular property, or theorem, affects the analysed signal or system. Usually these types of courses are accompanied with a set of laboratory exercises which tries to develop better understanding of course material. There have been many attempts to develop such a set (e.g. [1-10]). There are two most popular approaches. One is to use project based exercises, where students learn through experiments (such as [4,10-11]) based on the cliche "What I hear I remember, but what I do I understand", and the second is to use a PC computer and/or other hardware (usually audio and video stimulations) to help students to understand the course material. [2-3,5-6,7,9]. The first approach has proven to be better, but it requires more laboratory hours. The latter approach is cheaper, faster, and it can be rapidly adopted to students, if necessary.

As it is described in this paper, a set of laboratory exercises in the Signal Theory course is developed to encourage students without prior knowledge in signal processing and analysis to learn, understand, and experience most of the course topics in the most interesting way. The exercises, because of theirs 9-hour laboratory limit (based on ECTS credits for this course on Faculty of Electrical Engineering Sarajevo) and importance for future engineers of this department, are emphasizing these themes only:

- Interpretation and understanding of frequency spectrum and spectrogram of periodic and aperiodic signals
- Digital Signal time-domain compression and expansion
- Aliasing effect
- Understanding Fourier Series
- Interpretation of periodic signal spectrum (with aliasing effects)

These laboratory exercises are also held in the Signal and System Analysis course, as a part of the larger laboratory exercise set (14 hours).

The paper is divided and organized in accordance with listed topics. As it is shown in section Results and according to the faculty poll, students have shown better curriculum understanding, were more motivated and significantly pleased with laboratory exercises.

II. Interpretation and understanding of frequency spectrum

This laboratory exercise is the first students' encounter with practical interpretation of signals defined in time and frequency domain. Several time domain signals are introduced to students in order to familiarize with, using MATLAB as a programming environment. The analysed signals are essentialy discrete signals (because they are acquired and analysed in a discrete time domain - on a personal computer), but the whole concept of the laboratory exercise is adapted so that students believe these are analog signals $f(t)$. Students find it easily to conceptualize analog signals, because they hear (and see) them every day and their time-domain decription is almost natural to them. This should be a starting point for the exercise.

The set of chosen signals (such as the one given in Table I) should be demonstrated to students. The authors of this paper used the applet *xpsound* which can be found in MATLAB, and it is very illustrative (as shown in Fig. 1). All of the signals

TABLE I
TYPES OF SIGNAL ANALYSED - XPSOUND APPLET

Signal name	Description
Bird chirps	High frequency quasiperiodic signal
Chinese Gong	Low frequency signal
Hallelujah	Multiple voices cause almost flat freq.spectrum
Dropping Egg	Illustrate the Doppler effect
Train Whistle	Repetitive square wave tone
Laughter	Multiple voices + Delta functions
Beats	Constant sine tone
FM	Frequency modulated signal

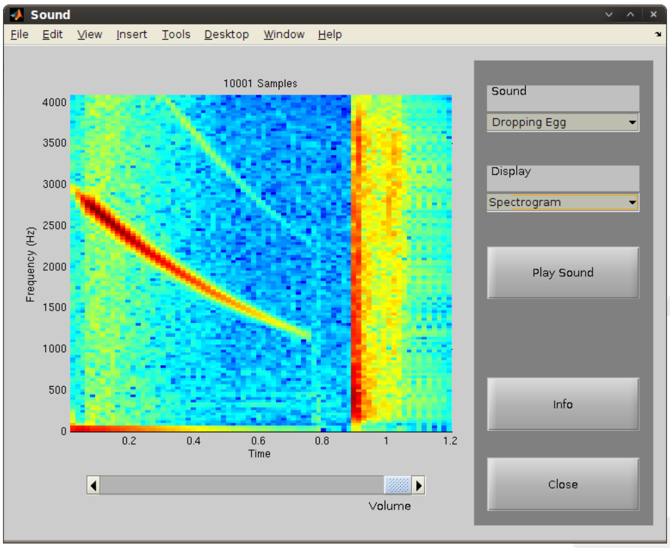

Fig. 1. Graphical interface of xpsound applet

from Table I can be visualized in time domain (and reproduced with speakers), and in frequency domain (using frequency spectrum and spectrogram). The frequency spectrum shows the decomposition of the signal into its frequency components, usually plotted as amplitudes of harmonics versus frequency [12], which is called amplitude spectrum. A spectrogram is an image of a time-varying spectral representation that shows how the spectral density of a signal varies with time [13].

It can be shown that analysis of these signals in time domain gives little or no additional informations about the heared sounds. But, once the students find out how to read and interprete frequency domain informations from the set of examples, it becomes interesting to investigate more properties of the analysed signals.

Many observations can be made from the analysed signals (given in Table I). Some of them are:

- Frequency ranges of different signals are different. By analysing the spectrum of human speech, it can be experimentally shown that frequency range lies under 3kHz (as shown in literature [14]), while birds' chirp occupie a higher frequency band (from 2 to 4 kHz). Students can experience (hear) higher tones (or frequencies), and visualize them (through spectrum and spectrograms), which is the first practical understanding of frequency spectrum of the signal.
- Spectrums of periodic signals are discrete functions. A continuous-time Fourier transform of a signal $f(t)$ is defined with:

$$F\{f(t)\} = \int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt \qquad (1)$$

where j is the imaginary unit, and $f(t)$ should satisfy certain conditions (Dirichlet Conditions etc., see e.g. [15]). After introducing the Dirac delta function with:

$$\delta(x) = \begin{cases} +\infty & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases} \qquad (2)$$

$$\int_{-\infty}^{+\infty} \delta(x)dx = 1 \qquad (3)$$

It can be shown that a Fourier transform of the sine function can be written as:

$$F\{Asin(\omega_0 t)\} = \frac{A\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] \qquad (4)$$

where $\delta(t)$ is the Dirac function. When first introduced with the Fourier transform (or even Fourier series) of the sine signal, students usually do not "believe" the applicability of relation (4), because delta function is defined with an unnatural description. But when they see the clear peak in frequency spectrum, or the clear line in the spectrogram, and hear the single constant sine tone of a certain frequency, they rapidly connect the line in frequency spectrum with the sine component, which creates an outstanding experience for them. Then it can be shown that spectrums of all common periodic functions (square, triangle, sawtooth wave etc.) are discrete.
- As can be shown, the Fourier transform of Dirac delta function is the constant in frequency domain:

$$F\{A\delta(t)\} = A \qquad (5)$$

This is also difficult mathematical concept for students to understand. This can be easily explained on spectrogram of various signals where the sudden impact is present (a gong hit, egg fall, snort etc.), as it is shown in Fig. 1. It is clear that all frequencies are present in the moment of the impact, and are equally represented in frequency spectrum.
- While analysing the gong hit on spectrogram, it can be easily shown that the gong acts like a lowpass filter, rapidly absorbing high-frequency components of the Dirac delta function. Also, the hit signal in time-domain can be interpreted as an impulse response of the gong.

During and after this laboratory exercise, students start to "feel" the frequency representations of signals. This happens mostly because other than the mathematical representation, students start to "hear" and "see" the frequency spectrum. Compared to only showing various mathematical interpretations, the presented approach results in the higher level of understanding. Many authors ([5-6]) also have recognized that multimedia assisted exercises increase the quality of understanding Signal Theory and its concepts. Other than that, many of the future engineers will work on real audio and video signals, so this and the following exercises try to prepare them for that as well.

III. Time-domain Discretization, compression and expansion of signals

The analog signal $f(t)$ in time-domain can be compressed or expanded in time domain. Usually, time domain compressed/expanded signal is represented by $f(at)$ where a is a real positive number. $f(at)$ is time-compressed $f(t)$ for $a > 1$, and time-expanded for $0 < a < 1$. As can be easily shown from (1), the expansion and compression in time domain create compression and expansion respectively in the frequency domain:

$$F\{f(at)\} = \frac{1}{a}F(j\frac{\omega}{a}) \tag{6}$$

where $F(j\omega)$ is the Fourier transform of $f(t)$.

In this laboratory exercise, the main goal is to reproduce (and hear) the compressed and expanded sound signal, and observe the spectrum in these cases.

Students should be first introduced to the concepts of discretization. An analog signal (human speech) should be recorded using MATLAB, and discretized using a chosen sampling frequency. It is wise to choose a $f_s = 22050Hz$ sampling frequency which is high enough to satisfy Nyquist-Shannon sampling theorem, but low enough to later reproduce correctly the signal with doubled sampling frequency, on a common PC sound card (44100 Hz). When acquired, the analog signal will be represented as discrete signal (vector with N elements), which means that every $T_s = 1/f_s$ seconds the sample has been acquired and stored as a part of sequence. If this signal is reproduced on standard speakers with sampling frequency of $f_s/2$, that would be equivalent to the expansion of signal in time domain $f(t/2)$. The latter experiment results in a deeper and slower voice, which is the direct consequence of equation (6). When reproducing the signal with sampling frequency of $2f_s$, the speech will be faster and higher in frequency spectrum (almost infantile and feminine), directly showing the expansion in frequency spectrum. Better students will observe that obviously male voices lie in lower frequency range than female and children voices. The students enthusiasm, and the smile on their faces (when they here the voices modified) will be guaranteed. Now the relation 6 also have its full physical meaning.

Also, in this laboratory exercise, a convolution of signals should be illustrated to students. The convolution is defined as [16]:

$$f(t) * h(t) = \int_{-\infty}^{+\infty} f(\tau)h(t-\tau)d\tau \tag{7}$$

The simplest way for students to understand convolution is by trying to create an echo-effect of the signal $f(t)$ from the voice they recorded. Students will quickly figure out what they have to achieve - to reproduce a sum of delayed and attenuated original signals, for example $f_{echo}(t) = f(t)+0.5f(t-0.3)+0.25f(t-0.5)$. It will soon be clear that echo can be formed by simple convolution of signals:

$$f_{echo}(t) = f(t) * g(t) \tag{8}$$

where $g(t)$ is defined as:

$$g(t) = \delta(t) + 0.5\delta(t-0.3) + 0.25\delta(t-0.5) \tag{9}$$

It is trivial to implement signal $g(t)$ in the discrete form - it is a vector full of zeros, with only several ones at places that correspond to 0, 0.3 and 0.5 seconds (1, $0.3f_s$ and $0.5f_s$ respectively). Even more interesting is that the signal that students hear is much like sound they hear on stadiums, or in public meetings. What they hear is the sum of several signals of different amplitude coming from speakers of different distances from the speaker. Experimenting with coefficients in (9) will create different signals. Also is interesting that students will interprete the relation (9) as an impulse response of a room in which the sound is analysed.

IV. Understanding sampling theorem and aliasing

The Nyquist-Shannon sampling theorem is one of the most fundamental theorems in the signal theory. The original formulation (by Shannon [17]) is "If a function contains no frequencies higher than W cps, it is completely determined by giving its ordinates at a series of points spaced 1/2 W seconds apart". There are many other formulations but generally it is defined as "A band limited, continuous time signal containing frequencies up to a maximum of F_{max} [Hz] may be represented with complete accuracy if it is sampled at a frequency of at least $2F_{max}$ [Hz]. This highest frequency is usually called the Nyquist frequency. However, if the signal is under-sampled (for e.g. at $F_{max}/2$)), the theorem is violated and some information will be lost. Under these circumstances, the frequencies beyond the Nyquist point in the continuous signal masquerade as low frequencies in the discrete version, being mirrored about the Nyquist point. Technically, this is referred to as aliasing.

Aliasing is a concept that students intiutively understand - when a signal is changing with some maximal rate, it should be sampled with a sampling frequency at least twice as fast. Most of them only memorize equations such as $f_s \geq 2F_{max}$, without going deeper into the meaning of it. As an author observed in [6], "when saying the word theorem, most students tend to fall asleep". Also, not many of them understand the physical meaning of aliased and complementary spectrums of the discretized signal.

In this laboratory exercise, students will be bored at start, while considering there is nothing new to learn. The exercise start by generating a sine tone with certain frequency (they could start with $f_{sin} = 200$ [Hz]), and reproduce it on PC speakers with a default frequency ($f_s = 44100$ [Hz]). Then, the students should gradually increase the frequency f_{sin} from 200Hz to 15kHz. They will hear a beep sound, which will increase its frequency, going into the higher pitch. On a 10-12 kHz frequency, most students will not hear anything. When

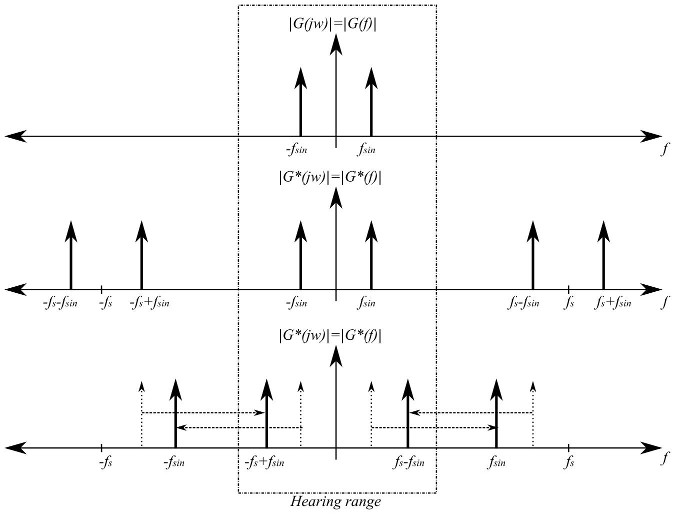

Fig. 2. Amplitude spectrum of aliased sine signals (amplitude scaling is neglected in this experiment for easier explaination). It is clear how higher frequency components are aliased into the hearing frequency range.

asked to continue increasing the frequency, some of them will ascertain that increasing frequency does not make sense, because they will continue to hear nothing. But, their curiosity will be increased, when they hear a sine tone on 35kHz. Most of them will try to give a meaningful explanation. The explanation which the tutor should give them are given on Fig. 2. The sine tone they hear is a consequence of aliasing, because the sampling theorem was not satisfied. Most of them have completely forgotten about the sampling frequency. Obviously, the sine frequency is greater then the half of the sampling frequency ($f_{sin} > f_s/2$). Students will be delighted, and aliasing will be fully understood. Some examples of aliasing from everyday life (spinning wheel which looks like turning backwards etc.) could be demonstrated.

V. Understanding Fourier Series

Experience and literature (e.g. [5] and [6]) show that the tecnical knowledge and skills are developed faster through competitive environment. In this laboratory exercise, students are divided into smaller groups (e.g. two or three students), competing with other teams (class tutor could suggest a knock-out game system). The main goal of this exercise is to fully understand the concepts of a Fourier Series of periodic functions. The Fourier Series is an expansion of a periodic function $f(t)$ in terms of an infinite sum of sines and cosines. There are many different representations of Fourier Series [15], but the most adequate way to present it to students is in the following form:

$$f(t) = A_0 + \sum_{k=1}^{+\infty} A_k sin(k\omega_0 t + \Phi_k) \qquad (10)$$

where A_i are amplitudes, and Φ_i phases of harmonics respectively, and can be calculated from function $f(t)$ over a defined interval (see e.g. [15] for details). The classical approach in presenting Fourier Series is adding sine functions in relation (10) from $k = 0$ to some constant, and presenting how it converges to a desired signal. This is often a boring, and dull approach for students.

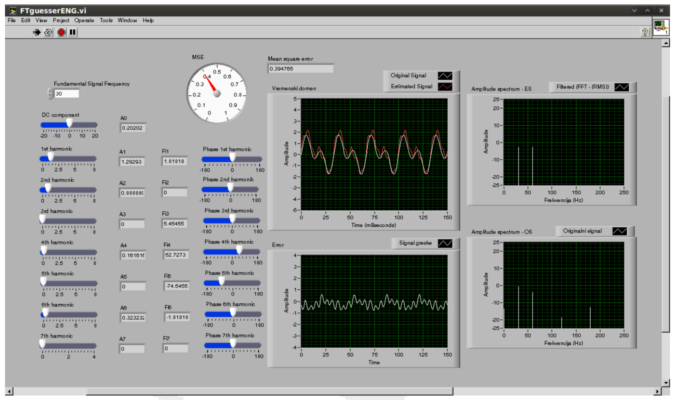

Fig. 3. LabView Front panel for FTGuesser.vi

Taking the latter into consideration, the following exercise is organized on a LabView Virtual Instrument model (whose front panel is shown on Fig. 3), where several standard test signals (square, triangle, sawtooth, sine etc.) of certain frequencies can be added to form a diverse composite periodic signal. It is important to set one signal frequency as a multiple of another signal frequency, because a finite number of higher-frequency harmonics can be generated, and they are obviously very dependent on a fundamental frequency.

Students' task is to approximate randomly (manually or automatically) generated signal as close as possible with the finite number of harmonics setting the amplitude and phase of every single sine harmonic. To aid students in this task, the time-domain representations of signals, spectrums of original and generated signal, time-domain error function and mean-square error can be viewed. Students can change the amplitude and phase of every single harmonic (up to 7th), and change the fundamental frequency.

There are two approaches. The first is that the tutor defines the same (random) signal to both teams, and the teams have to approximate the signal with smaller mean-square error in defined time (or under certain mean-square error in shorter time) than the other team. The other approach is that one team defines the signal to the other team. Here students learn even signal synthesis, and quickly find out which signals are more difficult to guess. The team with a smaller mean-square error in a defined time wins.

Usually a general strategy is to recognize the fundamental frequency, and then try to set the amplitudes of single harmonics so that frequency spectrums match as close as possible. It is important to notice that the signal is passed through a lowpass filter to eliminate aliasing on spectrums, so that it doesn't confuse the students. After that, students mostly need some luck to guess the phases of single harmonics. This is shown to be a good thing because it doesn't favor better students only. The experience has shown that weak students ussualy are harder to motivate in such games (quizzes, time-games etc.),

Fig. 4. Interactive laboratory - Students play in teams against each other

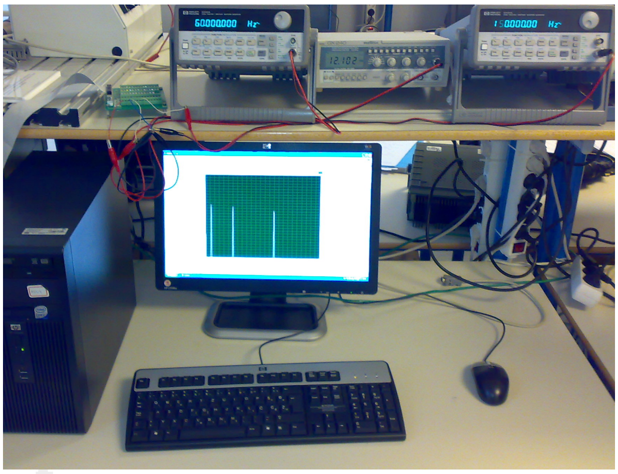

Fig. 5. Laboratory exercise Setup

because they often think their better colleagues are impossible to beat. In this laboratory exercise, all students have almost equal chances of winning, which is an additional motive for weak students to improve and learn more.

This laboratory exercise also developes a competitive spirit, which is important for future engineers productivity in the working environment.

Photo of one of the competitions is shown on Fig. 4.

VI. Interpretation of periodic signal spectrum (with aliasing effects)

The ability to correctly read frequency spectrums of discrete signals is one of the most important tools for signal processing engineers, hence should be clearly understood. Also, the limitations of Fast and Discrete Fourier Transform should be perfectly clear to students. Most authors usually use MATLAB or LabView ([4],[8-9]) to explain FFT to students. The simplest way to familiarize the students with the concept of the Fast Fourier Transform is by the spectral analysis of discrete signals. In this laboratory exercise, three signal generators (Fig. 5) are used for analog signal generation, and are connected to analog inputs of the data acquisition card (NI 6024E). The analog signals are sampled with frequency $f_s = 500$[Hz] and converted to 10bit digital values that can be used as inputs in the created LabView Virtual Instrument (VI). This VI is used as the spectrum analyser, which shows only the positive fundamental frequency band (from 0 to $f_s/2$), according to the Nyquist-Shannon sampling theorem.

Students have to prepare a signal (which can be formed as a sum of maximum three signals: sine, triangle, square of certain frequencies), to draw its spectrum, and to calculate FFT on $N = 1024$ samples (using MATLAB for example). The other students' teams in the group will then, only by observing the spectrum, try to guess the type and frequencies of added signals. These typical signals have their own characteristics - sine signal has only one peak in spectrum in the shown band (according to equation (4)), while triangle and square, having only odd harmonic components, differ by the gradient of decreasing of the amplitude of spectral components, which can be easily shown ([15]), and can be clearly seen on the spectrum. During the semester, students (even the weak ones) are familiarized with the concepts od spectrums and aliasing. Many of the students' homeworks (especially better ones) will favor square and triangle signals of 250Hz or 50Hz frequency, so they could mislead their colleagues in thinking there are only one or two sine components.

First thing the students learn from this laboratory exercise is that by only observing the spectrum, without any additional information, the given task is almost impossible to do. The other problem is aliasing - without prior signal filtering it is impossible to determine if there is aliasing or not.

But for the sake of the laboratory exercises, students can use some help:

- Students whose signal is to be guessed say how many signals they have added
- Students whose signal is to be guessed say in which range the frequency of components lie (depending of this hint the guessing can be difficult or easy)
- Students that are guessing the signals can tell the other group to move the signal generator frequency for every signal for +10Hz and -10Hz. This can be used as a great help to determine if aliasing is present (increasing frequency will move the peak to the right of the spectrum if there is no aliasing, or to the left if there is), and also can be used to determine if the spectrums of single signals are superpositioned one on another. Also, if only one peak is moving then it is a sine, otherwise it is square or triangle signal.

Our experience shows that the results were not so good at the beginning, but students easily "catched up" the game. Also, the difficulty of the game was determined by the informations on frequency range.

TABLE II
RESULTS OF THE POLL

Signal Theory course	2008/2009	2009/2010	2010/2011
The subject was well organized.	4.75	4.74	4.95
My previous knowledge was good enough to understand the subject content.	3.94	4.41	4.7
The subject was useful.	4.72	4.81	4.90
The subject stimulated my interest for this material.	4.72	4.78	4.89
Work procedures in the laboratory were well explained.	4.44	4.58	4.84
Laboratory work contributed better understanding of the material.	4.29	4.63	4.84
Size and complexity of laboratory tasks were correct.	4.23	4.52	4.61
Total grade of the subject.	4.59	4.65	4.90
Signals and Systems Analysis course	**2008/2009**	**2009/2010**	**2010/2011**
The subject was well organized.	4.67	4.62	4.88
My previous knowledge was good enough to understand the subject content.	4.43	4.54	4.58
The subject was useful.	4.62	4.54	4.92
The subject stimulated my interest for this material.	4.29	4.19	4.67
Work procedures in the laboratory were well explained.	4.48	3.76	4.54
Laboratory work contributed better understanding of the material.	4.43	4.12	4.54
Size and complexity of laboratory tasks were correct.	3.52	2.92	4.3
Total grade of the subject.	4.71	4.3	4.83

VII. DISCUSSION OF RESULTS, CONCLUSION, AND FUTURE WORK

Table II shows the results of the polls which are conducted on Faculty of Electrical Engineering Sarajevo every year. All students are surveyed (about 30-40 students per course every year), and the grades vary from 1 (the worst) to 5 (the best). The results of some questions on subject organisation and laboratory exercises are shown for the last 3 years. First year was conducted without the laboratory exercises described in this paper, the second year were partially introduced, and the last years the exercises were fully conducted in the way described in this paper. The results clearly show that students like the ”new” exercises more than the conventional ones.

It is difficult to illustrate pro objectivly the measures of the course material understanding, considering that the students are different every year. In our future work, we plan to divide students into two groups, one with the conventional exercises, and another with the novel laboratory exercises, so it could be proven that the ”through game” approach facilitate the understanding of course material. Also, more tests are going to be conducted, to objectively assess the degree to which the students are meeting the learning objectives. However, by refreshing laboratory exercises the subjective opinion of the teaching staff on the courses is that mostly because the mathematical relations could be heared, visualized, and experienced through many illustrative examples, students learn and understand the course material much better and get more familiarized with the fundamental concepts of the signal analysis and processing. Students also develop team work and competitive skills which they will find useful in the future professional activities.

It is planned to add and develop new laboratory exercises, which will include more audio and video processing, mostly for the Signal and Systems Analysis course. Image processing is also an interesting playground for students to learn basic signal processing of multidimensional signals, and it is fun to visualize different DSP concepts.

Another benefit of this type of exercises is that even students without much previous knowledge in Signal Theory or Mathematics will find them interesting. This could be useful for students of lower years, or even at technical college.

REFERENCES

[1] T. Bose, “A digital signal processing laboratory for undergraduates,” *IEEE Transactions on Education*, 1994.

[2] S. E. Budge, “A laboratory-based course in real-time digital signal processing implementation,” *Digital Signal Processing Workshop and 5th IEEE Signal Processing Education Workshop*, 2009.

[3] X. Chen and Y. Han, “Design and implementation of digital signal processing soft laboratory,” *Computer Science and Software Engineering, IEEE*, 2008.

[4] T. Sansaloni and et al, “FFT spectrum analyzer project for teaching digital signal processing with FPGA devices,” *IEEE Transactions on Education*, 2007.

[5] R. Browne, “Learning introductory signal processing using multimedia,” *IEEE Transactions on Education*, 2000.

[6] J. Pierre, R. Kubichek, and J. Hamann, “Reinforcing the understanding of signal processing concepts using audio exercises,” *Acoustics, Speech, and Signal Processing*, 1999.

[7] B. H. Ferri and et al., “Signal processing experiments with the LEGO MINDSTORMS NXT kit for use in signals and systems courses,” *American Control Conference*, 2009.

[8] F. Moumtadi, R. Mendoza, J. Delgado, and R. Medina, “Spectrum analyzer using FFT via LabVIEW,” *Electronics, Communications and Computer (CONIELECOMP)*, 2010.

[9] C. Burrus, “Teaching the FFT using matlab,” *Acoustics, Speech, and Signal Processing, IEEE*, 1992.

[10] S. Sridharan, V. Chandrana, and M. Dawson, “The design and development of an undergraduate signal processing laboratory,” *Acoustics, Speech, and Signal Processing, IEEE*, 1994.

[11] E. Sokic and M. Ahic-Djokic, “Simple computer vision system for chess playing robot manipulator as a project-based learning example,” *IEEE International Symposium on Signal Processing and Information Technology ISSPIT*, 2008.

[12] C. Alexander and M. Sadiku, *Fundamentals of Electric Circuits*. McGraw-Hill, 2004.

[13] S. Haykin, *Advances in Spectrum Analysis and Array Processing*. Prentice Hall, 1991.

[14] S. W. Smith, *The Scientist and Engineer’s Guide to Digital Signal Processing*. California Technical Pub., 1997.

[15] R. Bracewell, *The Fourier Transform and Its Applications*. McGraw-Hill, 1999.

[16] A. V. Oppenheim, A. S. Willsky, and S. Hamid, *Signals and Systems (2nd Edition)*. Prentice Hall, 1996.

[17] C. E. Shannon, “Communication in the presence of noise,” *Proceedings of IEEE*, 1998.