

$$\begin{aligned}
\text{a)} \quad & \overline{(A \vee \overline{B} \overline{C} \overline{D}) (\overline{A} \overline{D} \vee B (\overline{C} \vee A))} = \overline{A \vee \overline{B} \overline{C} \overline{D}} \vee \overline{\overline{A} \overline{D} \vee B (\overline{C} \vee A)} = \overline{A} \overline{\overline{B} \overline{C} \overline{D}} \vee \overline{\overline{A} \overline{D} B (\overline{C} \vee A)} = \\
& = \overline{A} (\overline{\overline{B} \vee \overline{C} \vee \overline{D}}) \vee (\overline{\overline{A} \vee \overline{D}}) (\overline{B \vee \overline{C} \vee A}) = \overline{A} (\overline{B \vee C \vee D}) \vee (\overline{A \vee D}) (\overline{B \vee \overline{C} \vee A}) = \\
& = \overline{A} (\overline{B \vee C \vee D}) \vee (\overline{A \vee D}) (\overline{B \vee \overline{A} C}) = \overline{A} (\overline{B \vee C \vee D}) \vee (\overline{A} \overline{B} \vee \overline{A} \overline{A} C \vee \overline{B} \overline{D} \vee \overline{A} \overline{C} \overline{D}) = \\
& = \overline{A} \overline{B} \vee (\overline{A} \overline{C} \vee \overline{A} \overline{C} \overline{D}) \vee \overline{A} \overline{D} \vee \overline{A} \overline{B} \vee \overline{B} \overline{D} = \overline{A} \overline{B} \vee \overline{A} \overline{C} \vee \overline{A} \overline{D} \vee \overline{A} \overline{B} \vee \overline{B} \overline{D} = \\
& = \overline{A} \overline{B} \vee \overline{A} \overline{C} \vee \overline{A} \overline{D} \vee \overline{A} \overline{B} \vee \overline{B} \overline{D} (\overline{A \vee A}) = \overline{A} \overline{B} \vee \overline{A} \overline{C} \vee \overline{A} \overline{D} \vee \overline{A} \overline{B} \vee \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} = \\
& = \overline{A} \overline{B} \vee \overline{A} \overline{C} \vee (\overline{A} \overline{D} \vee \overline{A} \overline{B} \overline{D}) \vee (\overline{A} \overline{B} \vee \overline{A} \overline{B} \overline{D}) = \overline{A} \overline{B} \vee \overline{A} \overline{C} \vee \overline{A} \overline{D} \vee \overline{A} \overline{B} = \overline{A} (\overline{B \vee C \vee D}) \vee \overline{A} \overline{B}
\end{aligned}$$

Teži dio ovog zadatka je eliminacija člana $\overline{B} \overline{D}$, što je učinjeno na sličan način kao u Primjeru 6.8 u udžbeniku. Ukoliko se dopusti i upotreba ekskluzivne disjunkcije, moguće je dobiti još kraći oblik ovog logičkog izraza na sljedeći način:

$$\overline{A} \overline{B} \vee \overline{A} \overline{C} \vee \overline{A} \overline{D} \vee \overline{A} \overline{B} = (\overline{A} \overline{B} \vee \overline{A} \overline{B}) \vee \overline{A} (\overline{C} \vee \overline{D}) = (A \oplus B) \vee \overline{A} (\overline{C} \vee \overline{D})$$

$$\begin{aligned}
\text{b)} \quad & \overline{\overline{A} \overline{B} \overline{C} \vee (\overline{A} \vee \overline{B} \vee \overline{D}) (\overline{A} \overline{B} \overline{D} \vee \overline{B})} = \overline{\overline{A} \overline{B} \overline{C}} \vee \overline{(\overline{A} \vee \overline{B} \vee \overline{D}) (\overline{A} \overline{B} \overline{D} \vee \overline{B})} = \\
& = (\overline{\overline{A} \vee \overline{B} \vee \overline{C}}) (\overline{\overline{A} \vee \overline{B} \vee \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee \overline{B}}) = (\overline{A \vee B \vee C}) (\overline{\overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee \overline{B}}) = \\
& = (\overline{A \vee B \vee C}) (\overline{\overline{A} \overline{B} \overline{D} \vee (\overline{A} \vee \overline{B} \vee \overline{D}) \overline{B}}) = (\overline{A \vee B \vee C}) (\overline{\overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \vee \overline{B} \overline{B} \vee \overline{B} \overline{D}}) = \\
& = (\overline{A \vee B \vee C}) (\overline{\overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \vee \overline{B} \overline{D}}) = \\
& = \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{A} \overline{B} \vee \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{B} \vee \overline{B} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{B} \overline{C} \overline{D} = \\
& = \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{B} \overline{C} \overline{D} = \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee (\overline{A} \overline{B} \overline{C} \vee \overline{A} \overline{B} \overline{C} \overline{D}) \vee \overline{B} \overline{C} \overline{D} = \\
& = \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{B} \overline{C} \overline{D} = \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{B} \overline{C} \overline{D} (\overline{A \vee A}) = \\
& = \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{A} \overline{B} \overline{C} \overline{D} \vee \overline{A} \overline{B} \overline{C} \overline{D} = \overline{A} \overline{B} \overline{D} \vee (\overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \overline{D}) \vee (\overline{A} \overline{B} \overline{C} \vee \overline{A} \overline{B} \overline{C} \overline{D}) = \\
& = \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C}
\end{aligned}$$

Do oblika $\overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{B} \overline{C} \overline{D}$ se sasvim lako dolazi. Interesantniji i teži dio ovog zadatka je eliminacija člana $\overline{B} \overline{C} \overline{D}$, što je također učinjeno na sličan način kao u Primjeru 6.8 u udžbeniku. Uz upotrebu ekskluzivne disjunkcije, moguće je izvršiti još neka pojednostavljenja:

$$\overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} = A (\overline{B} \overline{D} \vee \overline{B} \overline{D}) \vee \overline{A} \overline{B} \overline{C} = A \overline{B} \oplus \overline{D} \vee \overline{A} \overline{B} \overline{C}$$

Napomena: Da smo slučajno umjesto izraza $\overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{B} \overline{C} \overline{D}$ dobili izraz u kojem je zadnja pojava promjenljive D negirana (tj. izraz $\overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{B} \overline{C} \overline{D}$), dalji postupak pojednostavljivanja bi bio znatno manje očigledan. S obzirom da se u tom postupku koriste neke interesantne transformacije koje zaista nisu očigledne na prvi pogled, ovdje će radi ilustracije nekih korisnih ideja biti prikazane i taj postupak:

$$\begin{aligned}
& \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{B} \overline{C} \overline{D} = \overline{A} \overline{B} \overline{D} \vee (\overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \overline{D}) \vee \overline{A} \overline{B} \overline{C} \vee (\overline{B} \overline{C} \overline{D} \vee \overline{A} \overline{B} \overline{C} \overline{D}) = \\
& = \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee (\overline{A} \overline{B} \overline{C} \overline{D} \vee \overline{A} \overline{B} \overline{C} \overline{D}) \vee \overline{A} \overline{B} \overline{C} \vee \overline{B} \overline{C} \overline{D} = \\
& = \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee (\overline{A} \overline{B} \overline{C} \vee \overline{A} \overline{B} \overline{C}) \vee \overline{B} \overline{C} \overline{D} = \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee (\overline{B} \overline{C} \vee \overline{B} \overline{C} \overline{D}) = \\
& = \overline{A} \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{D} \vee \overline{B} \overline{C} = A (\overline{B} \overline{D} \vee \overline{B} \overline{D}) \vee \overline{B} \overline{C} = A \overline{B} \oplus \overline{D} \vee \overline{B} \overline{C}
\end{aligned}$$

Naravno, specijalni metodi razvijeni za potrebe minimizacije kao što su Quineov algoritam i Veitchovi dijagrami, koji se obrađuju u poglavljima 8. i 9. u udžbeniku, oslobađaju nas potrebe za pronalaženjem ovakvih neočiglednih “ad hoc” transformacija i omogućavaju da se postupak minimizacije provede rutinski.